EFFICIENT APPROXIMATIVE METHOD OF COMPUTING CONIFER STEM VOLUME

Pero Radonja, Ljubinko Rakonjac, Mihailo Ratkovic
Institute of Forestry - Belgrade

Abstract

An efficient approximative method for calculation of the cumulative conifer stem volume was developed. The method is based on the approximation of stem profile function with the linear function, generated by linear fitting of the available data. To verify the study results, they were compared to cumulative volume obtained when the profile function corresponds to modified Brink’s function. It is known that the application of modified Brink’s function enables the calculation of practically real volumes. The mean deviation in the application of approximative procedure is less than 2%, which justifies the use of proposed procedure. The developed volume models, i.e. the cumulative volume functions based on linear function and modified Brink’s function consist of 3, i.e. 6 components respectively. The cumulative volume functions show the saturation, because with the approaching to total stem height, volume increment decreases with the decrease of radius. The course of the cumulative volume function of approximative method in its last fourth is approaching to the course of real cumulative volume function and thus enables the low deviation of the calculated volume from real volume.

Key words: approximative method, modelling of volume, stem profile, linear fitting

INTRODUCTION

Stem volume is a significant datum for analyses of the success and profitability of the established stand. Therefore, this datum is very important also in the selection of tree species for afforestation and melioration.

Volume calculation can be approached in several ways, so there are numerous different methods, first of all the stereometric and physical ones. The latter are used only exceptionally, when the volume has to be very precisely calculated for the stems of extremely irregular shapes. Generally, when high accuracy is necessary, usually the modified Brink’s function is applied for the approximation of the morphological curve of the conifer stem (Riemer et al., 1995).

On the other hand, sometimes we are satisfied with the approximative volume data. We shall present an efficient approximative method for the calculation of conifer stem volume.
The classical method of stem volume calculation is based on finding a relation or a set of relations that ensure an adequate accuracy of volume calculation. The model that ensures an adequate accuracy reported in literature is the segmented polynomial model by Max and Burkhart (Max, Burkhart, 1976). Three segments, i.e. domains are differentiated in the papers (Avery, Burkhart, 1994; Kozak, 1988). Mirković and Banković (Mirković, Banković, 1993) use different values of the parameter $r$ for individual parts of the stem in the general equation of morphological curve. The initial step is calculation of the defined integral, where the limits of the integral are tree heights and sub-integral function is the square of profile function, i.e. the square of function representing the morphological curve in given case. This study, instead of the division into segments, deals with a unique morphological curve in the definition of volume model. Essentially, it is the calculation of volume of a figure generated by the rotation of morphological curve around the $x$-axis.

**MODELS OF THE CUMULATIVE VOLUME**

When stem volume is calculated under the hypothesis that we deal with the regular solid figures, the accuracy of the results is frequently unacceptably low. On the other hand, if it is taken that individually observed stems are mainly asymmetric and of irregular shape due to the environmental impact on the modification of the regular growth and formation, the relations are too complex and inapplicable in practice. To obtain the satisfactory relations for practical application, the dendrometry takes into account the symmetric solid figures. In that case, volume $V(X)$ can be obtained as the defined integral from 0 to a height $X$, of the solid figure generated by the rotation of the profile function $y(x)$ around the $x$-axis, i.e. by using the following relation, (Mirković, Banković, 1993):

$$V(x) = \pi \int_0^x y^2(x) \, dx$$  \hspace{1cm} (1)

We shall first analyse the models of cumulative volume where the modelling of profile function is based on the linear function obtained by the procedure of linear fitting of available data. The proposed approximative method of volume calculation is actually based on such models. To verify the obtained results, we shall also analyse the results obtained when the modified Brink's function is used as the profile function. Along with the volume function, in both cases their components will also be presented. The analysis of the components is important from the aspect of selection and comparison of the potential different approximations. The analysis of the components also enables defining the corrective component in the potential addition of such a component in the aim of increasing the accuracy of volume calculation.

**Proposed approximative method based on linear fitting of the data**

Volume calculation in the case of linear function of stem profile is a trivial problem from the mathematical aspect. However, we shall present the function of the cumulative volume, as well as its components, aiming at comparison with the results referring to the case when profile function corresponds to modified Brink's function.

The linear function of stem profile obtained by using the procedure of linear fitting of the data has the following form:

$$y(x) = n - kx$$  \hspace{1cm} (2)

The coefficients $n$ and $k$ are determined by the procedure of fitting. The total volume is calculated by applying the relation (1) and the definite integral is calculated to

$$X = \frac{n}{k}$$  \hspace{1cm} (3)

It should be noted that we do not use real stem height or radius at breast height. Fig. 1 presents the real data for a 50-year-old spruce, at the altitude of 1160 m. It also shows a straight line obtained by the procedure of linear fitting, which is equivalent to the real profile function, regarding the volume. Mean error of fitting is 0.6859 cm.

The function of cumulative volume is obtained based on relations (1) and (2). If the required calculations are done, the components of the volume function have the following forms:

$$V'_1(x) = \pi n^2 x$$  \hspace{1cm} (4)

$$V'_2(x) = -\pi k n x^2$$  \hspace{1cm} (5)

$$V'_3(x) = \frac{k^2}{3} x^3$$  \hspace{1cm} (6)

- where evidently:

$$V(x) = \sum_{i=1}^{3} V'_i(x)$$  \hspace{1cm} (7)

The volume components $V'_1(x)$, $V'_2(x)$ and $V'_3(x)$ versus the stem height are presented in Fig. 2. The first component is represented by a solid line, and the second component is represented by a dashed line, for $x = X$ they have identical values, only with an opposite sign. The third component is represented by a dotted line.

The function of the cumulative volume based on the relation (7), presented in Fig. 3, is calculated by the program in MATLAB, "MATLAB" (2000). It actually shows the volumes of the successive truncated cones, where the height at which the cones are truncated is gradually increased. The volume of the complete cone $x = X$ is the
approximation volume of calculated conifer stem. The following section will present comparison of volumes calculated by proposed approximative procedure with the volumes calculated by the procedure which can be considered as very precise, according to the literature data (Riemer et al., 1995), (Radonja et al., 2005).

Volume model by the application of modified Brink's function

As it has been mentioned, this section deals with the analytical expression of the model of cumulative stem volume by using the modified Brink's function as the profile function, Fig. 4. The mean error of fitting i.e. modelling 0.0939 cm is, as expected, considerably lower than by linear fitting, Fig. 1. The sub-integral function is the square of the canonical form of the modified Brink's function, (Riemer et al., 1995):

\[ V(x) = \pi \int (u + ve^{-px} - ve^{qx})^2 \, dx \quad (8) \]

If the necessary calculation is done, the components of the volume model have the following forms:

\[ V_1(x) = \pi u^2 x \quad (9) \]

\[ V_2(x) = \pi \frac{w^2}{2p} (1 - e^{-2px}) \quad (10) \]

\[ V_3(x) = -\pi \frac{w^2}{2q} (1 - e^{2qx}) \quad (11) \]

\[ V_4(x) = \pi \frac{2uw}{p} (1 - e^{-px}) \quad (12) \]

\[ V_5(x) = \pi \frac{2uw}{q} (1 - e^{qx}) \quad (13) \]

\[ V_6(x) = -\pi \frac{2uw}{p-q} (1 - e^{(q-p)x}) \quad (14) \]

where \( V(x) \) is evidently the sum of all the presented components.

As the odd components are considerably greater than even components, in the aim of distinction, Fig. 5 presents separately only the odd components. The first component \( V_1(x) \) is a linear component and it is represented by a solid line. Component \( V_3(x) \) marked by a dashed line and the component \( V_5(x) \) marked by a dotted line, are functions of the exponent \( q (e^{+^q} > f) \). For the low values of the heights, \( x < X/2 \), the component \( V_3(x) \) and \( V_5(x) \) are practically linear. This results from the fact that the value of \( q \) usually lies from 0.03 to 0.10. In the global (general) analysis, for the sake of the simplicity, the value of \( q \) is usually taken as 0.1.
Fig. 6 presents the even components of cumulative volume function, i.e. the components whose form primarily depends on the value of the parameter $p$.

The value of parameter $p$ is usually within the limits 0.8 to 1.7. In the rough analysis, aiming at a simpler comparison and analysis of the results, it is usually taken that the value of $p$ is 1. In the components $V_2(x)$ and $V_4(x)$, only parameter $p$ is explicit. In the component $V_6(x)$ both $p$ and $q$ are explicit, however as $p>>q$, practically only the effect of parameter $p$ is expressed. The component $V_2(x)$ is marked by a solid line. The component $V_4(x)$ is marked by a dashed line, and the component $V_6(x)$ is marked by a dotted line.

The components $V_2(x)$, $V_4(x)$ and $V_6(x)$, Fig. 6, are obviously nonlinear, but only for small heights, up to 20% of the total stem height. In the study case, it is up to the height of 5 m. After that height, all the three components are practically invariable.

The course of the cumulative volume function, $V(x)$, is presented in Fig. 7. The function is saturated, which is understandable, as with increased height the stem diameter is decreased, so volume increment becomes lower. Also, the course of the cumulative volume function by the approximative method, Fig. 3, in its last fourth is approaching to the course of the real cumulative volume function, Fig. 7. This produces a small deviation of the volume calculated by the approximative method, from the real volume.

**STUDY RESULTS – ANALYSIS OF THE PROPOSED APPROXIMATIVE METHOD ACCURACY**

The accuracy of the proposed approximative method is tested by 6 spruce trees originating from the highest altitude, 1300 m. The 16-year-old tree originates from the lowest altitude of 550 m. There is a regularity of the attained heights, so that the oldest tree reached the superior height of 29.7 m, and the lowest height of 5.65 m was attained by the youngest 12-year-old tree, Table 1.

The comparison of the volumes calculated by the approximative method and by more precise procedure (based on modified Brink's function as the stem profile function) shows that the approximative method produces mainly somewhat greater volumes. Only in one case the volume calculated by the approximative method (0.0136) was lower than the more precisely calculated volume 0.0138 m³.

The highest deviation, i.e. error, accounts for 2.39% and the lowest is about 1%, more precisely, 0.998%. The correct results, the error of about 1.5%, are calculated in the case of approximately uniformly distributed measurements. Certainly, if we have grouped measurements, e.g. 3, referring to practically the same stem height, the error can be lower, but also higher than 2%. Such a series of measurements, in the statistical sense, does not give a regular series of data. For the trees presented in Table 1, the mean deviation, i.e. the error of calculated volume by proposed approximative method is 1.58%.

**CONCLUSION**

It is known that very precise method of calculation of stem volume is based on the use of modified Brink's function as the function of stem profile. However, in this case, the concrete modified Brink's function must be computerised, which means the input of about 125 characters, i.e. numbers, brackets or symbols of arithmetic operations. Also, the expression for volume calculation has 6 components i.e. members.

The proposed method based on the application of linear fitting does not require the introduction of a special function i.e. relation, and also the expression for the volume calculation has only three components. The application of the proposed approximative method gives the volume with the average error less than 2%. The study result justifies
Table 1
Accuracy of the approximative method

<table>
<thead>
<tr>
<th>Spruce Age (years)</th>
<th>127</th>
<th>53</th>
<th>30</th>
<th>16</th>
<th>14</th>
<th>12</th>
</tr>
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<tbody>
<tr>
<td>Height (m)</td>
<td>29.70</td>
<td>18.70</td>
<td>13.20</td>
<td>6.30</td>
<td>5.80</td>
<td>5.65</td>
</tr>
<tr>
<td>Radius at breast height (cm)</td>
<td>21.6</td>
<td>10.4</td>
<td>6.7</td>
<td>3.5</td>
<td>3.0</td>
<td>4.4</td>
</tr>
<tr>
<td>Altitude (m)</td>
<td>1300</td>
<td>1160</td>
<td>990</td>
<td>550</td>
<td>635</td>
<td>800</td>
</tr>
<tr>
<td>Volume by Approximative method (m³)</td>
<td>1.8389</td>
<td>0.3035</td>
<td>0.1012</td>
<td>0.0136</td>
<td>0.0102</td>
<td>0.021</td>
</tr>
<tr>
<td>Volume by Modified Brink’s function (m³)</td>
<td>1.8142</td>
<td>0.2997</td>
<td>0.1002</td>
<td>0.0138</td>
<td>0.0100</td>
<td>0.0208</td>
</tr>
<tr>
<td>Error by the approximative method (%)</td>
<td>1.36</td>
<td>1.27</td>
<td>0.998</td>
<td>1.45</td>
<td>2.00</td>
<td>2.39</td>
</tr>
</tbody>
</table>

Acknowledgements: This study is financed by the Ministry of Science and Environmental Protection of the Republic of Serbia in the framework of the project Selection of tree species for afforestation and reforestation, (TR-6821).

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Ijubinko Rakonjac: e-mail: ljubinko_rakonjac@eunet.yu
Mihailo Ratkovic: e-mail: mratkovic@yube.net